Lecture 8: main exercises

Exercise 8.1. For each of the following braid words
(a) Draw its 3D plabic graph. (The answer is on the back of this page, but don't look yet!!)
(b) Draw the soap films for some solid crossings. You should use the graphs on the back of this page.

(i) $\beta = \sigma_1^2\sigma_2^3\sigma_1^2\sigma_2\sigma_1 \in \mathrm{Br}_3^+$.
(ii) $\beta = \sigma_3\sigma_2\sigma_1\sigma_3^2\sigma_1\sigma_2\sigma_1\sigma_2\sigma_3^2\sigma_1\sigma_2 \in \mathrm{Br}_4^+$.

Based on these examples, what distinguishes the soap films S_d for mutable Deodhar hypersurfaces hypersurface from those for frozen Deodhar hypersurfaces?

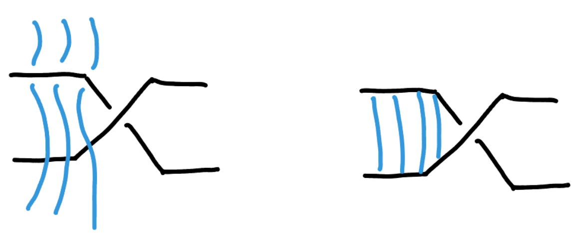
Exercise 8.2. Let j be a hollow crossing, and let $\Delta^{\uparrow}$, $\Delta^{\downarrow}$, $\Delta^{\rightarrow}$, $\Delta^{\leftarrow}$ be the grid minors above, below, to the right, and to the left of crossing j in the 3D plabic graph. Use propagation rules to show that

$$\frac{\Delta^{\uparrow}\Delta^{\downarrow}}{\Delta^{\rightarrow}\Delta^{\leftarrow}} = 1.$$

Lecture 8: additional exercises

Exercise 8.3. Prove that the hollow crossings of a braid word β form the leftmost subexpression for w_0 in β. To be more precise, any other subexpression for w_0 will consist of a set of crossings that is lexicographically larger than the set of hollow crossings.

Exercise 8.4. The propagation rules implicitly assert that soap films can always "pass through" hollow crossings undisturbed. That is, you never see either of the local configurations below.



Prove that these configurations never occur using the skew-shape lemma.