Lecture 5: main exercises

A quiver is called **isolated** if there are no arrows between mutable vertices.

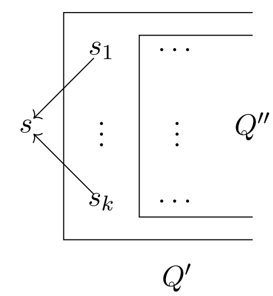
Exercise 5.1. Let $\mathcal{A}$ be an isolated cluster algebra.

(a) Describe all cluster variables (in all clusters) in $\mathcal{A}$.

(b) Describe $\mathcal{A}$ by generators and relations.

The class of **sink-recurrent quivers** is defined recursively as follows. Assume first that Q has no frozen vertices.

- Any isolated quiver Q is sink-recurrent.
- Any quiver that is mutation equivalent to a sink-recurrent quiver is sink-recurrent.
- Suppose that a quiver Q has a sink vertex s, let $N(s) = \{s_1, \dots, s_k\}$ be the set of neighbors of s (note that the arrows from s_i to s could have multiplicities). If the quivers $Q' = Q - \{s\}$ and $Q'' = Q - \{s\} - N(s)$ are sink-recurrent, then Q is sink-recurrent.



More generally, Q is sink-recurrent if its mutable part is sink-recurrent.

Exercise 5.2. (a) Assume that Q has a sink vertex s as above, and X is the corresponding cluster variety. Prove that

$$X = \{x_s \neq 0\} \cup \{x_{s_1} \cdots x_{s_k} \neq 0\}.$$

Hint: Assume first that there are no frozen vertices as in the figure above.

(b) Assume that Q is sink-recurrent. Prove that the corresponding cluster algebra is locally acyclic.

Lecture 5: additional exercises

Exercise 5.3. Suppose Q is an acyclic quiver. Prove that every edge between mutable vertices in Q is separating.

Exercise 5.4. (a) Prove that any acyclic cluster variety can be covered by isolated cluster charts.
Hint: use Exercise 5.3

(b) Prove that any locally acyclic cluster variety can be covered by isolated cluster charts.