

Lecture 14: main exercises

Exercise 14.1. Suppose

$$B_i(z_1)\chi_i(u_1)B_{i+1}(z_2)\chi_{i+1}(u_2)B_i(z_3)\chi_i(u_3) = B_{i+1}(z'_1)\chi_{i+1}(u'_1)B_i(z'_2)\chi_i(u'_2)B_{i+1}(z'_3)\chi_i(u'_3)$$

where all u, u' are invertible. Prove that z'_1, z'_2, z'_3 are uniquely determined by z_1, z_2, z_3 and $u_1, u_2, u_3, u'_1, u'_2, u'_3$ provided that

$$u_1u_2 = u'_2u'_3 \text{ and } u_2u_3 = u'_1u'_2.$$

Exercise 14.2. Assume that $\tilde{z}_2 \neq 0$. Prove that

$$B_i(z_1)\chi_i(u_1)B_i(z_2)\chi_i(u_2) = B_i(z)\chi_i(u)U$$

where $z = z_1 - u_1^{-2}\tilde{z}_2^{-1}$, $u = z_2u_1u_2$ and U is an upper unitriangular matrix.

Exercise 14.3. Compute the two pairs of cluster variables for two Demazure weaves from σ^3 to σ using problem 2. Verify that these are regular functions and that two seeds are related by mutation.

Hint: you will need to push an upper-triangular matrix to the right for one of the weaves.



