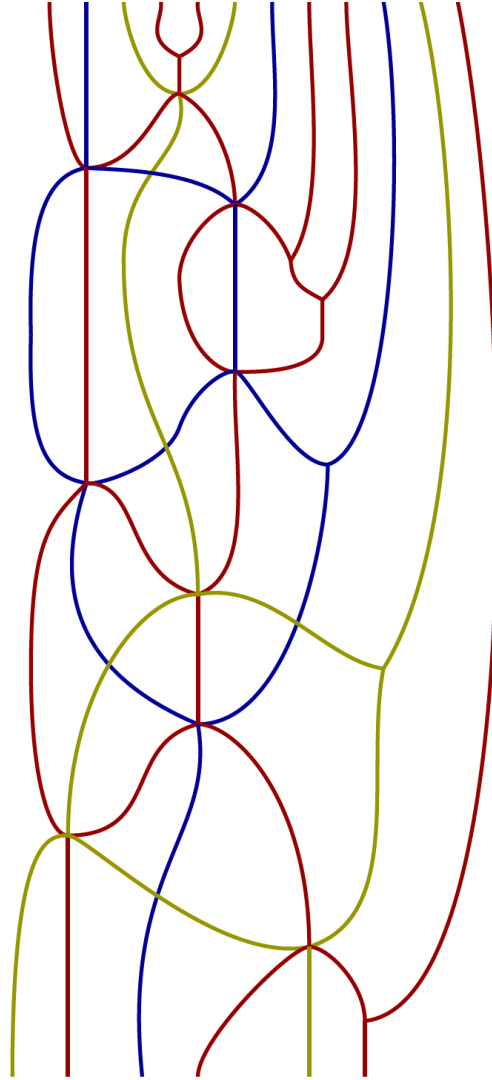
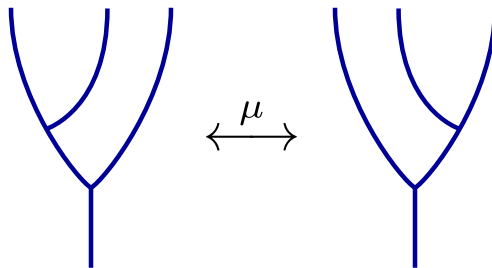


Lecture 13: main exercises

Exercise 13.1. In the weave below, identify all Lusztig cycles and compute their pairwise intersections.



Exercise 13.2. Suppose that two weaves $\mathfrak{W}, \mathfrak{W}'$ agree everywhere except for a weave from σ^3 to σ :



Let C_1 (resp. C'_1) be a Lusztig cycle which starts above this fragment and has values a, b, c on top edges. Let C (resp. C') be a short cycle supported on the internal edge. Prove $C_1 \cdot C = -C'_1 \cdot C'$ by computing both sides.

Lecture 13: additional exercises

Exercise 13.3. A **semifield** is a set with addition $\oplus$, multiplication $\otimes$ and division (but no subtraction) satisfying the usual axioms.

- (a) Prove that $\mathbb{Z} \cup \{\infty\}$ with operations $a \oplus b = \min(a, b)$ and $a \otimes b = a + b$ is a semifield (sometimes called tropical semifield). Write a formula for division in this semifield.
- (b) Prove that the set R of rational functions $p(t)/q(t)$ where $p(t), q(t)$ are polynomials with nonnegative integer coefficients, and usual operations, is a semifield.
- (c) Prove that there is a unique semifield homomorphism $\nu : R \rightarrow \mathbb{Z} \cup \{\infty\}$ such that $\nu(t^a) = a$.