

Lecture 12: main exercises

Exercise 12.1. One tool in the proof that $\mathbb{C}[X(\beta)]$ is a cluster algebra is *rotation*. Suppose $\beta = \gamma i$ for γ a positive braid word on $[n]$. Define $i^* := n - i$ and let $\beta' = i^* \gamma$. Construct an isomorphism $X(\beta) \rightarrow X(\beta')$.

Exercise 12.2. Suppose $\beta = \gamma ii$ and say β has r letters. Suppose $r - 1 \notin J_\beta$, that is, the second-to-last i is hollow.

- (a) Express x_r , the cluster variable associated to the final letter of β in terms of grid minors.
- (b) Construct an isomorphism $X(\beta) \rightarrow X(\gamma i) \times \mathbb{C}^\times$ so that the coordinate of $\mathbb{C}^\times$ is x_r .
- (c) Verify that if you remove vertex r from Q_β and variable x_r from the cluster $\mathbf{x}_\beta$, you obtain the seed $\Sigma_{\gamma i}$.

