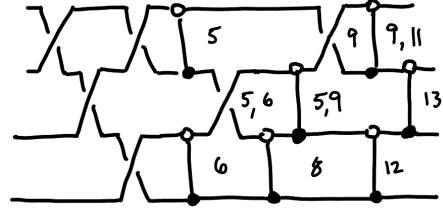
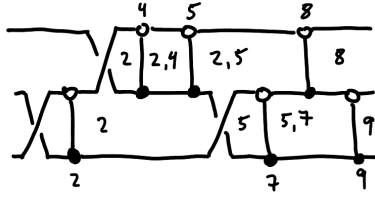


Lecture 11: main exercises

Exercise 11.1. For each of the following braid words, draw the quiver Q_β .

- (i) $\beta = \sigma_1^2 \sigma_2^3 \sigma_1^2 \sigma_2 \sigma_1 \in \text{Br}_3^+$.
- (ii) $\beta = \sigma_3 \sigma_2 \sigma_1 \sigma_3^2 \sigma_1 \sigma_2 \sigma_1 \sigma_2 \sigma_3^2 \sigma_1 \sigma_2 \in \text{Br}_4^+$.

To make this a little faster, here are the 3D plabic graphs, with each region labeled by the cluster variables that appear there.



Exercise 11.2. Suppose β, β' are related by a braid move, that is $\beta = \dots iji \dots$ and $\beta' = \dots jij \dots$ where $|i - j| = 1$. Convince yourself of the following.

- (a) If any letters involved in the braid move are hollow, then Q_β and $Q_{\beta'}$ are related by relabeling.
- (b) If all letters involved in the braid move are solid, then $Q_{\beta'}$ is a relabeling of $\mu_r(Q_\beta)$, where the leftmost letter in the braid move is letter r of β .

Lecture 11: additional exercises

Exercise 11.3. Show that the half-arrow and surface description of the quiver agree. Hint: For the surface description, if you force the curves C_d to lie on the graph G_β , then $C_d \cap C_{d'}$ is a disjoint union of paths. The signed intersection number of the curves can be computed one path at a time.