Math 432/532 (Diff. Top.), Winter 2016
HW 5

Starred problems are for 532 students, and are extra credit for 432 students (not worth as much as a usual problem). 532 students must LaTeX their solutions.

To give a problem practicing the formalities of proper maps, I'm recalling this definition which will come in handy again and again.

Definition 0.1. A *fiber bundle* $\pi\colon E \to B$ with *fiber* F is a map π of topological spaces with the following property: B has a cover by open sets such that for each open U in this cover, $\pi^{-1}(U) \cong U \times F$, with π being projection to the first factor.

That is, fiber bundles are maps that, locally on the base, are product maps. For example $E = B \times F$, mapping by projection to B, is a fiber bundle, but a boring one. The Mobius strip E is a fiber bundle over $B = S^1$ with fiber $F = (-1, 1)$, but is not homeomorphic to the cylinder $S^1 \times (-1, 1)$. The tangent space TX of an n-manifold X is a fiber bundle over X with fiber $\mathbb{R}^n$.

1. Show that a fiber bundle $\pi\colon E \to B$ with fiber F is proper if and only if F is compact.
2. G+P Chapter 1.4 exercises 3, 4.
3. G+P Chapter 1.5 exercises 1, 2. Undergrads can skip 2g.
4. G+P Chapter 1.5 exercise 5. Undergrads may opt to do exercise 4 instead.
5. G+P Chapter 1.5 exercise 7, 8.
6. G+P Chapter 1.5 exercise 9*, 10*. (Hint on 10: consider two different submanifolds of $X \times X$. Are they transverse?)
7. G+P Chapter 1.5 exercise 11*.
8. (EXTRA CREDIT) Partial converse 1, on p24 of G+P, is false as stated; it is only true locally, and the proof they give is local. Find a counterexample.