Math 432/532 (Diff. Top.), Winter 2016
HW 4

Starred problems are for 532 students, and are extra credit for 432 students (not worth as much as a usual problem). 532 students must LaTeX their solutions.

1. Consider S^2 as the sphere of radius one in $\mathbb{R}^3$. Let $F\colon S^2 \to S^2$ be the map which rotates the sphere one quarter turn around the north-south pole. That is, both the north and south poles are fixed; when viewed from the north pole looking toward the center of the earth, the equator is rotated "clockwise" by $\pi/2$ radians.
   (a) Explicitly describe the map dF_p for $p = (1, 0, 0)$. That is, choose a basis of T_pS^2, and say where it goes with respect to a basis of $T_{F(p)}S^2$.
   (b) Explicitly describe the map dF_N for the north pole $N = (0, 0, 1)$. This is a fixed point of F, so choose a basis of T_NS^2, and write dF_N as a matrix.
2. (*) Repeat the exercise above for the map $G\colon S^2 \to S^2$ which sends $(x, y, z) \mapsto (-x, y, z)$.
3. G+P Chapter 1.3 exercise 8.
4. G+P Chapter 1.4 exercises 1, 2.
5. (*) G+P Chapter 1.4 exercise 7.
6. Prove that, for an immersion, each fiber (i.e. preimage of some point) has the discrete topology. Find a smooth map between manifolds which is not an immersion, where the fibers have the discrete topology. Find a smooth map which is not an immersion where exactly two fibers do not have the discrete topology.
7. (*) G+P Chapter 1.4, exercise 8. Ultimately, this is a question about a map $\mathbb{R}^2 \to \mathbb{R}^2$, but this may not be the easiest perspective. (You'll want to recall from complex analysis what it means to be holomorphic.)
8. Let X and Y be manifolds. Prove that the projection map $X \times Y \to X$ is a submersion. Prove that for any smooth function $F\colon X \to Y$, the graph $X \to X \times Y$, $x \mapsto (x, F(x))$ is an immersion.