Math 432/532 (Diff. Top.), Winter 2016
HW 3

Starred problems are for 532 students, and are extra credit for 432 students (not worth as much as a usual problem). 532 students must LaTeX their solutions.

1. G+P Chapter 1.2, exercise 1, 2, 4. These are warmups, they do not deserve more than two sentences each.
2. G+P Chapter 1.2, exercises 7, 8, 11. (You should also read 9, 10.)
3. (*) G+P Chapter 1.2, exercise 12.
4. G+P Chapter 1.4, exercise 5,6.
5. G+P Chapter 1.4, exercise 12. Which leads naturally to:
6. (*) Let M_n denote the vector space of $n \times n$ matrices. Recall that $M_n \cong \mathbb{R}^{n^2}$, with coordinates x_{ij} which correspond to the matrix with a 1 in the i-th row and j-th column, and zeroes everywhere else. Let $f\colon M_n \to \mathbb{R}$ be the determinant map.
   (a) Compute $\partial f/\partial x_{ij}$ for each i, j. (Hint: use the row or column expansion formula for determinants. If that hint doesn't make sense, try $n = 2$ and $n = 3$ until you see the pattern.)
   (b) What is df? (This should be a matrix with one row and n^2 columns). If A is an invertible matrix, is df_A always surjective?
   (c) Deduce that $SL_n = f^{-1}(1)$, the set of all matrices with determinant 1, is a manifold of dimension $n^2 - 1$.
   (d) Analogous to the computation at the bottom of page 22 of G+P, prove that $df_I(B) = \mathrm{Tr}(B)$, where I is the identity matrix, and $\mathrm{Tr}(B)$ is the trace of B.
   (e) Check that the tangent space to SL_n at I consists of all matrices with trace equal to zero.