Homework 5

Due Wednesday, February 23, 2011

1. Let $X = S^2/\{(\pm 1, 0, 0)\}$ be the sphere with the north and south poles glued together. Compute $\pi_1(X)$.

2. Let X be a torus minus an open disc.

   (a) Argue that X is homotopy equivalent to $S^1 \vee S^1$.

   (b) From part (a), $\pi_1(X)$ is the free group on two generators a and b. Let ∂X be the boundary of X, which is a circle, $i : \partial X \to X$ the inclusion, and c a generator of $\pi_1(\partial X) = \mathbb{Z}$. What is $i_*(c)$?

   (c) Let Y be the two-holed torus obtained by gluing two copies of X together along their boundary circles. (This operation of taking two surfaces, cutting a disc out of each, and gluing the boundary circles is called the "connected sum.") Draw a picture. Use van Kampen's theorem to compute $\pi_1(Y)$ from this description.

   (d) What is the Abelianisation of the group in part (c)?

3. Use van Kampen's theorem to show that $\pi_1(S^n) = 0$ for $n \geq 2$. Why does the same argument not show that $\pi_1(S^1) = 0$?