Homework 4

Math 487

Due October 25, 2013

1. Our symbol set consists of a single binary relation symbol $<$. Let Φ be the axioms of a dense linear order without endpoints:

$$\forall x \forall y \, \neg(x \equiv y \wedge x < y)$$
$$\forall x \forall y \forall z \, ((x < y \wedge y < z) \rightarrow x < z)$$
$$\forall x \forall y \, (x < y \vee x > y \vee x \equiv y)$$

$$\forall x \forall y \, (x < y \rightarrow \exists z (x < z \wedge z < y))$$

$$\forall x \exists y \, (x < y)$$
$$\forall x \exists y \, (y < x).$$

   Show that any two countable models of Φ are isomorphic, following the "back-and-forth" argument we outlined in class.

2. Show that the theory $\Phi^{\models}$ is complete, that is, for every sentence φ we have either $\Phi \models \varphi$ or $\Phi \models \neg\varphi$.

Since $\Phi^{\models}$ is (finitely) axiomatizable and complete, it is decidable. Group theory, in contrast, is finitely axiomatizable and undecidable; hence it is incomplete, but we already knew that from looking at $\forall x \forall y \, x \circ y \equiv y \circ x$.