First Midterm

Name: _____

February 19, 2013

1. (a) Write an equation for the plane containing the line $x = 1 + t$, $y = 1 - t$, $z = 0$ and the point $(0, 0, 1)$. Check that the line and the point satisfy the equation you come up with.

   (b) Write an equation for the plane containing the same line and the point $(2, 2, 1)$.

   (c) Find the cosine of the angle between the planes in parts (a) and (b).

2. The shape of a power line hanging between two telephone poles (or any other hanging cable or chain) is not a parabola as you might guess, but a *catenary*, described by the equation

$$y = \frac{e^x + e^{-x}}{2}.$$

Parametrize this as

$$\mathbf{r}(t) = \langle t, \tfrac{e^t + e^{-t}}{2} \rangle$$

and find the length of the curve between $t = -1$ and $t = 1$. Hint: The thing under the square root can be factored; look at the thing you expanded out to get an idea for how to factor it.

3. Consider the surface $y = x^2 + z^2$.

   (a) Sketch the slices $x = 0$, $x = 1$, and $x = 2$.

   (b) Sketch the slices $y = 0$, $y = 1$, and $y = 2$.

   (c) Sketch the surface. The slices you drew in parts (a) and (b) should appear.

4. Find the limit, or show that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{x^2+y^2}$.

(b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y^2}{x^2+y^2}$.

5. Find the maximum and minimum values of the function $f(x, y) = x^2 + x + y^2 - y$ on the disc $x^2 + y^2 \leq 1$.

6. Use linear approximation to estimate the value of $\sqrt{(.9)(2.9)(3.1)}$. Hint: The true value is $2.84446831587205\ldots$