

Correciton to our tangential and normal acceleration example

Math 212

January 24, 2013

In class today we had

$$\vec{v} = \langle 1, 2, 3 \rangle$$
$$\vec{a} = \langle 0, 2, 6 \rangle$$

$$\vec{a} \cdot \vec{v} = 1 \times 0 + 2 \times 2 + 3 \times 6 = 22$$

$$|\vec{v}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$|\vec{a}| = \sqrt{0^2 + 2^2 + 6^2} = \sqrt{40} = 2\sqrt{10}$$

$$a_T = \frac{\vec{a} \cdot \vec{v}}{|\vec{v}|} = \frac{22}{\sqrt{14}}$$

$$\vec{T} = \frac{\vec{v}}{|\vec{v}|} = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

$$\vec{a}_{\parallel} = a_T \vec{T} = \frac{\vec{a} \cdot \vec{v}}{v \cdot v} \vec{v} = \frac{11}{7} \langle 1, 2, 3 \rangle = \left\langle \frac{11}{7}, \frac{22}{7}, \frac{33}{7} \right\rangle$$

$$\vec{a}_{\perp} = \vec{a} - \vec{a}_{\parallel} = \left\langle -\frac{11}{7}, -\frac{8}{7}, \frac{9}{7} \right\rangle$$

$$a_N = |\vec{a}_{\perp}| = \frac{\sqrt{11^2 + 8^2 + 9^2}}{7} = \frac{\sqrt{266}}{7} = \frac{\sqrt{38}}{\sqrt{7}}$$

and on the other hand

$$a_N^2 = |\vec{a}|^2 - a_T^2 = 40 - \frac{22^2}{14} = 40 - \frac{484}{14} = 40 - \frac{242}{7} = \frac{38}{7},$$

so we were doing fine right up until the very end. Sorry I made a mess of it.
Yet a third way to calculate a_N is:

$$\vec{a} \times \vec{v} = \langle 6, -6, 2 \rangle = 2\langle 3, -3, 1 \rangle$$

$$|\vec{a} \times \vec{v}| = 2\sqrt{3^2 + 3^2 + 1^2} = 2\sqrt{19}$$

$$a_N = \frac{|\vec{a} \times \vec{v}|}{|\vec{v}|} = \frac{2\sqrt{19}}{\sqrt{14}} = \frac{\sqrt{38}}{\sqrt{7}}.$$