Second Midterm

Name: ______________________________

November 13, 2013

1. (20 points) Find the maximum and minimum of the function $f(x, y) = x^2y^2$ subject to the constraint $x^2 + y^2 = 3$. Hint: If you divide through by x, also consider the possibility that $x = 0$.

2. (a) (10 points) Find the critical points of the function $f(x, y) = (\sin x)(2 + \sin y)$ in the region $0 \leq x < 2\pi$, $0 \leq y < 2\pi$. Hint: There are four.

(b) (10 points) For each critical point, decide whether it is a local maximum, a local minimum, or a saddle point. Hint: Each occurs at least once.

3. (a) (5 points) Sketch the plane region lying above the parabola $y = x^2$ and below the line $y = 1$.

(b) (5 points) Find the area of the region.

(c) (10 points) Find the centroid of the region. Hint: For the y-coordinate you need to do a double integral, but for the x-coordinate you don't.

4. (a) (5 points) Sketch the region in space lying above the paraboloid $z = x^2 + y^2$ and below the plane $z = 1$.

(b) (5 points) Find the volume of the region. Hint: The thing is round. If you want to use rectangular coordinates that's your business, but if you do I won't give partial credit.

(c) (10 points) Find the centroid of the region. Hint: For the z-coordinate you need to do a triple integral, but for the x- and y-coordinates you don't.

(d) (moral points) Is the centroid of this region higher or lower than that of the plane region in the previous problem? Is that reasonable?

5. (a) (5 points) Sketch the region R in the first quadrant that lies between the curves $y = 1 - x^2$ and $y = 3(1 - x^2)$.

(b) (5 points) Consider the change of coordinates $x = u, \quad y = v(1 - u^2)$. What do the region's bounds become in these coordinates?

(c) (5 points) Find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$.

(d) (5 points) Evaluate $\displaystyle\iint_R \frac{x^2}{y}\, dx\, dy$ using this change of coordinates.