

Worksheet 9

Math 634, Algebraic Topology

Monday, October 19, 2020

In the category of sets, every surjection has a section: if X and Y are sets and $f: X \rightarrow Y$ is a surjection, then there is an $s: Y \rightarrow X$ with $f \circ s = 1_Y$. This is the axiom of choice.

But the same statement is not true in the category of topological spaces. Let $f: S^1 \rightarrow S^1$ be given by $f(z) = z^2$. (Draw a picture.) Observe that this is a continuous surjection. Show that there is no continuous map $s: S^1 \rightarrow S^1$ with $f \circ s = 1$. Hint: Consider the induced maps on π_1 .