Worksheet 19

Math 634, Algebraic Topology

Monday, November 23, 2020

In lecture we contemplated the diagrams

$$\begin{tikzcd}
 & \vdots \arrow[d] & \vdots \arrow[d] & \vdots \arrow[d] & \\
0 \arrow[r] & C_n(A) \arrow[r, "i_\sharp"] \arrow[d, "\partial"'] & C_n(X) \arrow[r, "q"] \arrow[d, "\partial"'] & C_n(X,A) \arrow[r] \arrow[d, "\partial"'] & 0 \\
0 \arrow[r] & C_{n-1}(A) \arrow[r, "i_\sharp"] \arrow[d] & C_{n-1}(X) \arrow[r, "q"] \arrow[d] & C_{n-1}(X,A) \arrow[r] \arrow[d] & 0 \\
 & \vdots & \vdots & \vdots &
\end{tikzcd}$$

and

$$\cdots \longrightarrow H_n(A) \xrightarrow{i_*} H_n(X) \xrightarrow{q_*} H_n(X,A) \xrightarrow{\partial} H_{n-1}(A) \xrightarrow{i_*} H_{n-1}(X) \longrightarrow \cdots .$$

We proved some of the following facts. Work on proving the others. Be sure to draw enough pictures.

0. ∂ is a well-defined homomorphism.
1. $\operatorname{im} i_* \subset \ker q_*$.
2. $\ker q_* \subset \operatorname{im} i_*$.
3. $\operatorname{im} q_* \subset \ker \partial$.
4. $\ker \partial \subset \operatorname{im} q_*$.
5. $\operatorname{im} \partial \subset \ker i_*$.
6. $\ker i_* \subset \operatorname{im} \partial$.

Together these prove that the second diagram is a long exact sequence.