Worksheet 17

Math 634, Algebraic Topology

Monday, November 16, 2020

For a map $f\colon X \to Y$, we defined induced maps $f_\sharp$ as in the commutative diagram

$$\begin{tikzcd}
\cdots \arrow[r] & C_{n+1}(X) \arrow[r, "\partial"] \arrow[d, "f_\sharp"'] & C_n(X) \arrow[r, "\partial"] \arrow[d, "f_\sharp"'] & C_{n-1}(X) \arrow[r] \arrow[d, "f_\sharp"'] & \cdots \\
\cdots \arrow[r] & C_{n+1}(Y) \arrow[r, "\partial"] & C_n(Y) \arrow[r, "\partial"] & C_{n-1}(Y) \arrow[r] & \cdots ,
\end{tikzcd}$$

and induced maps $f_*\colon H_n(X) \to H_n(Y)$. We showed that if $f \simeq g$ then there are maps

$$P\colon C_n(X) \to C_{n+1}(Y)$$

such that

$$f_\sharp - g_\sharp = P\partial + \partial P.$$

Show that if there is such a map P then $f_* = g_*$: that is, if $z \in C_n(X)$ is a cycle, then $f_\sharp(z)$ and $g_\sharp(z)$ represent the same class in $H_n(Y)$.