Worksheet 16

Math 634, Algebraic Topology

Friday, November 13, 2020

In lecture we defined the standard n-simplex

$$\Delta^n = \left\{(x_0, \dots, x_n) \in \mathbb{R}^{n+1} : x_i \geq 0,\ \sum x_i = 1\right\},$$

the face maps $\varphi_i \colon \Delta^{n-1} \to \Delta^n$ given by

$$\varphi_i(x_0, \dots, x_{n-1}) = (x_0, \dots, x_{i-1}, 0, x_i, \dots, x_{n-1}),$$

and for an n-simplex $\sigma \colon \Delta^n \to X$, the boundary

$$\partial\sigma = \sum_{i=0}^{n} (-1)^i \sigma \circ \varphi_i \in C_{n-1}(X),$$

which we extended linearly to give a homomorphism

$$C_n(X) \to C_{n-1}(X).$$

1. Verify that if $i < j$ then $\varphi_j \circ \varphi_i = \varphi_i \circ \varphi_{j-1}$.
2. Use this to verify that $\partial\partial\sigma = 0$.