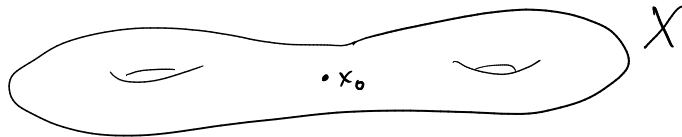


Midterm 2

Due Friday, November 20, 2020

You may refer to Hatcher's book, the lecture notes, and the homework problems and solutions, and you may quote any results proved there. But do not consult your colleagues or the internet.

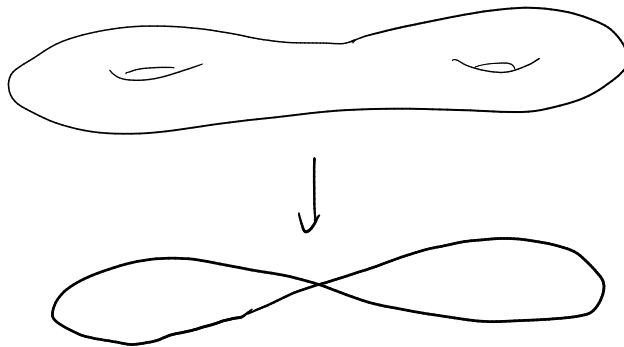
1. Let X be the orientable surface of genus 2, and let x_0 be a point in the middle of the "neck:"



- (a) Compute $\pi_1(X, x_0)$ by cutting X into two punctured tori and applying van Kampen's theorem.

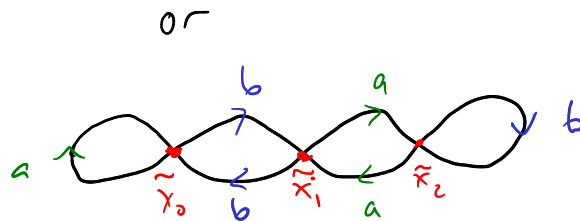
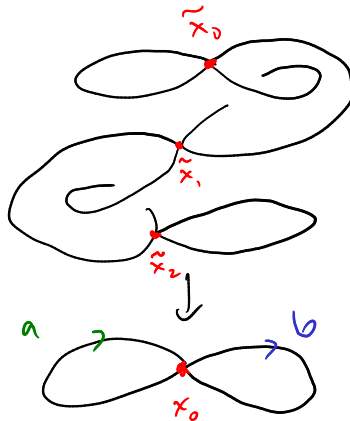
Hint: You analyzed the punctured torus on the second homework.

- (b) Show that X is not homeomorphic to the sphere S^2 or the torus $S^1 \times S^1$, as follows. Inspired by this picture,



write down a surjection from $\pi_1(X, x_0)$ onto the free group on two letters. We know that the latter is not Abelian, so $\pi_1(X, x_0)$ is not Abelian.

2. Let $X = S^1 \vee S^1$, let $x_0 \in X$ be the wedge point, let $p: \tilde{X} \rightarrow X$ be the three-sheeted cover shown below, and let $p^{-1}(x_0) = \{\tilde{x}_0, \tilde{x}_1, \tilde{x}_2\}$ as shown.



Let $\varphi: \tilde{X} \rightarrow \tilde{X}$ be a deck transformation, that is, a homeomorphism with $p \circ \varphi = p$.

- (a) Show that we cannot have $\varphi(\tilde{x}_0) = \tilde{x}_1$ or $\varphi(\tilde{x}_0) = \tilde{x}_2$.

Hint: One approach is via unique path lifting.

- (b) Conclude that φ is the identity.

Hint: The unique lifting property.