Homework 3

Due Monday, October 19, 2020

1. Hatcher, Chapter 0 #11.
2. Let (X, x) be a pointed space. Show that the reduced suspension ΣX is homeomorphic to the smash product $X \wedge S^1$.
3. Show that if $n \neq 0$, then the map $f\colon S^1 \to S^1$ given by $f(z) = z^n$ is not nullhomotopic.

   Hint: From our discussion of $\pi_1(S^1, 1)$, we know that the path $\gamma\colon I \to S^1$ given by $\gamma(x) = e^{2\pi i n x}$ and the constant path at 1 are not homotopic rel. endpoints. Together with Worksheet 5 #1, this shows that f and the constant path at 1 are not homotopic rel. the basepoint 1. How can you upgrade this to "not homotopic, even if we ignore basepoints"?
4. Draw pictures of each of the following spaces:
   (a) The sphere S^2 union the line segment connecting the north and south poles.
   (b) The wedge sum $S^2 \vee S^1$.
   (c) The sphere S^2 with the north and south poles glued together.
   (Actually there are two interesting ways to draw this embedded in $\mathbb{R}^3$: the gluing could happen inside or outside the sphere. Draw them both.)
   (d) The torus $S^1 \times S^1$ with a disc D^2 glued in along the circle $S^1 \times \{1\}$.
   (Again, draw it in two different ways.)

   Argue (loosely) that all four are homotopy equivalent, because you can get between them by collapsing some nice contractible subspaces. Later we'll try to pin this down more rigorously.