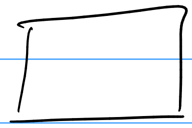
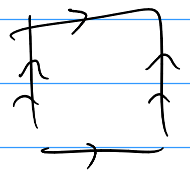
Worksheet from Friday:

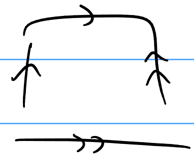


$\binom{4}{2} \cdot 2^4 = 96$ ways to put arrows on the edges.

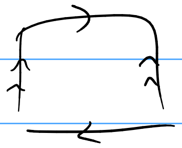
$D^4 \times \mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/2$ acts. Six orbits



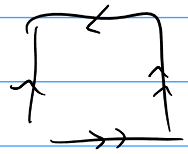
torus



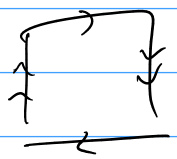
again torus!



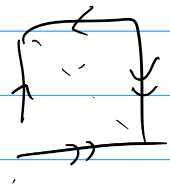
Klein bottle



again $\mathbb{RP}^2$



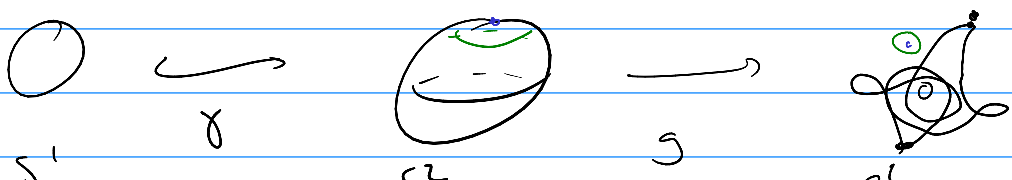
$\mathbb{RP}^2$ (proj. plane)



Sphere S^2

Homotopy

On first day we had maps



$$S^1 \xrightarrow{\gamma} S^2 \xrightarrow{g} \mathbb{R}^2 \setminus 0$$

We wanted to deform $g \circ \gamma$ by deforming γ to a const map

$I = [0,1]$

Def. Let $f_0, f_1 : X \to Y$

A *homotopy* from f_0 to f_1 is a map

$$F : X \times I \longrightarrow Y$$

with
$$F(x,0) = f_0(x) \quad \forall x \in X$$
$$F(x,1) = f_1(x) \quad \forall x \in X$$

example: $F : S^1 \times I \longrightarrow S^2$

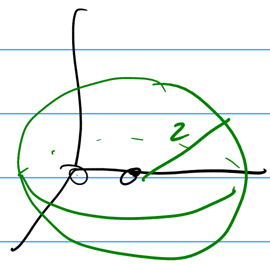
$$((x,y),t) \longmapsto \left(\sqrt{1-t^2}\,x,\ \sqrt{1-t^2}\,y,\ t\right)$$

We say that f_0, f_1 are *homotopic* if $\exists$ a homotopy between them.
Write $f_0 \simeq f_1$ TeX: \simeq

Example: The maps

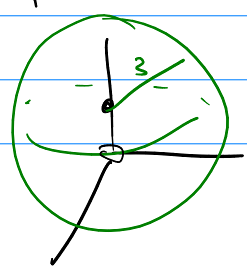
$$f_0 : S^2 \longrightarrow \mathbb{R}^3 \setminus 0$$

$$x \longmapsto 2x + (1,0,0)$$



and $f_1 : S^2 \longrightarrow \mathbb{R}^3 \setminus 0$

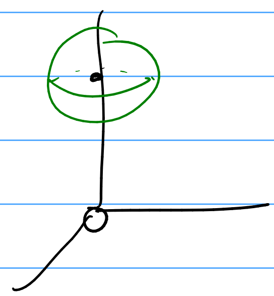
$$x \longmapsto 3x + (0,0,1)$$



are homotopic via the straight-line htpy

$$F(x,t) = (1-t)f_0(x) + t f_1(x)$$

but $g: S^2 \to \mathbb{R}^3 \setminus 0$
$x \longmapsto x + (2,0,0)$



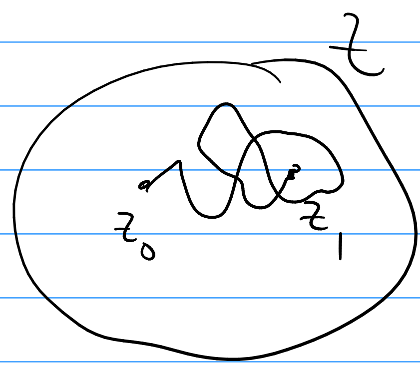
is not homotopic to f_0 and f_1.

it is htpic to a constant map.

Def: $f: X \to Y$ is <u>nullhomotopic</u> if it's homotopic to a constant map.

Second perspective on htpy: a path of maps.

A <u>path</u> in a space Z from z_0 to z_1 is a map $\gamma: I \to Z$ with $\gamma(0) = z_0$ $\gamma(1) = z_1$



Given $f_0, f_1 : X \to Y$ and a homotopy $F : X \times I \to Y$ between them,

define $f_t : X \to Y$ by

$$f_t(x) = F(x,t) \qquad \forall t \in I$$

This seems to be a path

$$\gamma : I \longrightarrow Z := \{\text{cont maps } X \to Y\}$$
$$t \longmapsto f_t$$

a path from f_0 to f_1.

Thm: if X is loc. compact
$\exists$ a top. on Z
s.t. γ is cont iff F was cont.

Pf: read about the compact-open top.