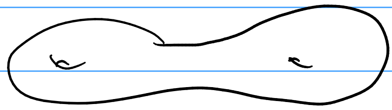
What Spaces Do We Care About?

① Manifolds — spaces where every point has a nbd. homeo. to $\mathbb{R}^n$

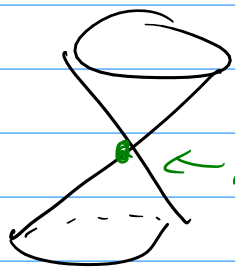
e.g. surfaces 

spheres $S^n = \{ \vec{x} \in \mathbb{R}^{n+1} : |\vec{x}| = 1 \}$ 

projective spaces $\mathbb{RP}^n$ and $\mathbb{CP}^n$ — soon.

② Algebraic Varieties — subsets of $\mathbb{R}^n$ or $\mathbb{C}^n$ (or $\mathbb{RP}^n$ or $\mathbb{CP}^n$) cut out by polynomial eqs.

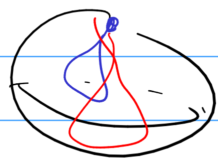
e.g. $\{ (x,y,z) \in \mathbb{R}^3 : x^2 + y^2 = z^2 \}$



← not a manifold at this point

③ Certain spaces of maps, e.g. loop spaces

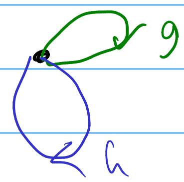
$\Omega S^2 = \{ \gamma : [0,1] \longrightarrow S^2 \mid \gamma(0) = \gamma(1) = \text{n. pole} \}$



(4) Simplicial complexes – combinatorial objects built out of triangles.

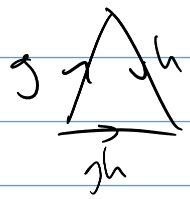
e.g. if G is a group, the <u>classifying space</u> BG:

start with a point

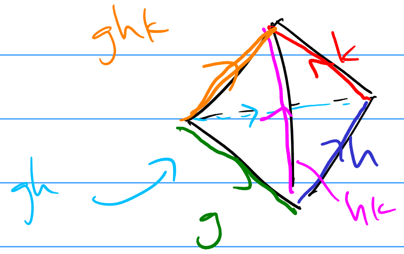


$\forall g \in G$, add an edge

$\forall g, h \in G$ glue in a triangle



$\forall g, h, k \in G$, glue in a tetrahedron

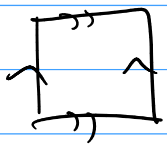


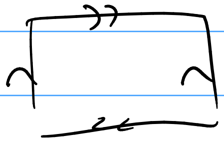
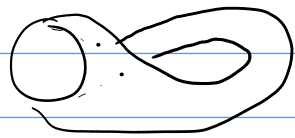
keep going.

top. of BG tells us about G.

(4) Don't care much about pathologies – e.g. spaces that are path-connected + locally connected but not loc. path-conn.

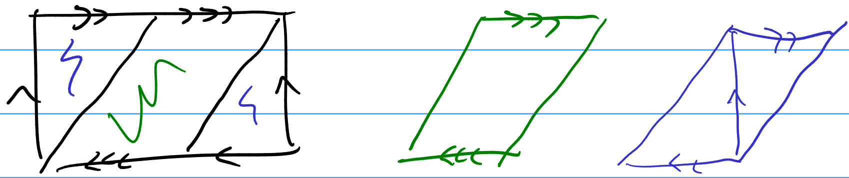
Some quotients of rectangles

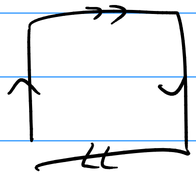
   torus.

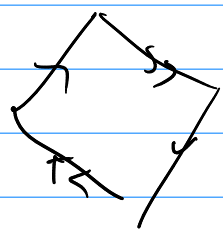
   Klein bottle

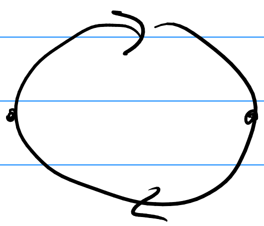
German: flasche = bottle
fläche = surface

Klein bottle = two Möbius strips glued along boundary circle.

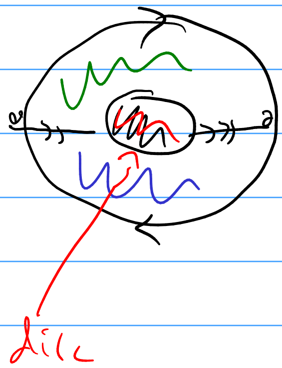


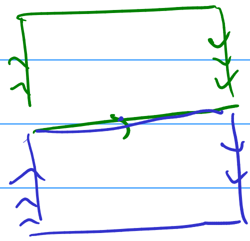
 projective plane $\mathbb{RP}^2$

Could also draw as 

or 

it's a Möbius strip glued to a disc along boundary circle:





Möbius strip: