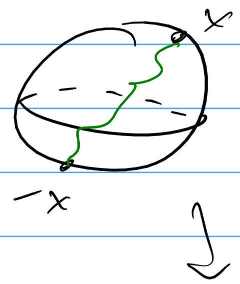
Math 634: first day

Claim 1: Let $S^2 = \{ \vec{x} \in \mathbb{R}^3 \mid |\vec{x}| = 1 \}$



and let $f: S^2 \to \mathbb{R}$ be continuous

then $\exists\, x \in S^2$ with $f(x) = f(-x)$.

Idea: consider $g(x) = f(x) - f(-x)$

$$g: S^2 \longrightarrow \mathbb{R}$$

seek x with $g(x) = 0$

suppose $g(x) > 0$ for some x

then $g(-x) = -g(x) < 0$

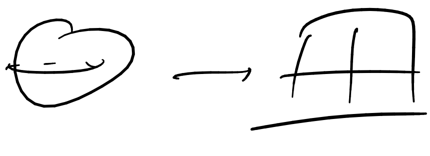
somewhere in between, must cross zero

by the intermediate value thm.

On worksheet: write this nicely.

Claim 2: Let $f: S^2 \to \mathbb{R}^2$ be continuous.

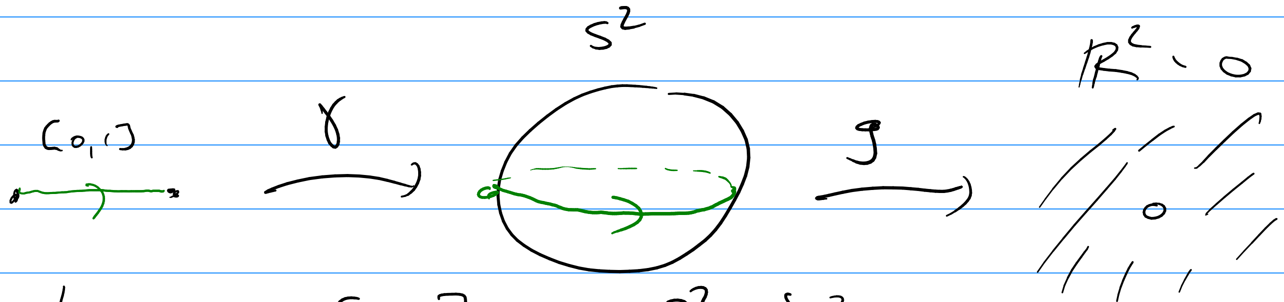
Then $\exists\, x \in S^2$ with $f(x) = f(-x)$



Idea: Again consider $g(x) = f(x) - f(-x)$

Suppose g is never zero, produce a contradiction.

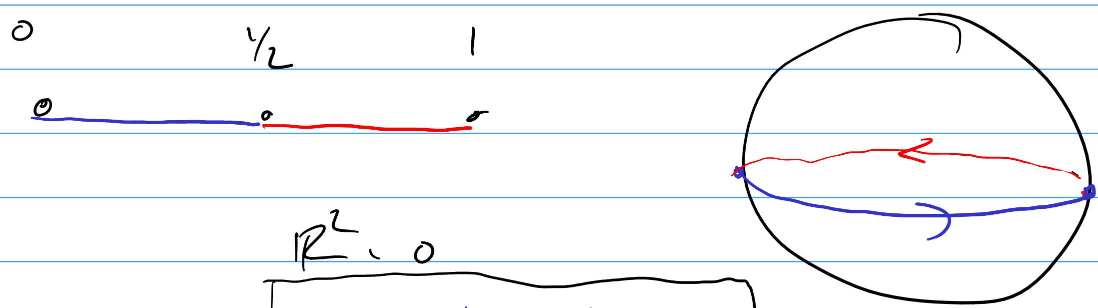
Let $\gamma : [0,1] \longrightarrow S^2$ parametrize the equator.

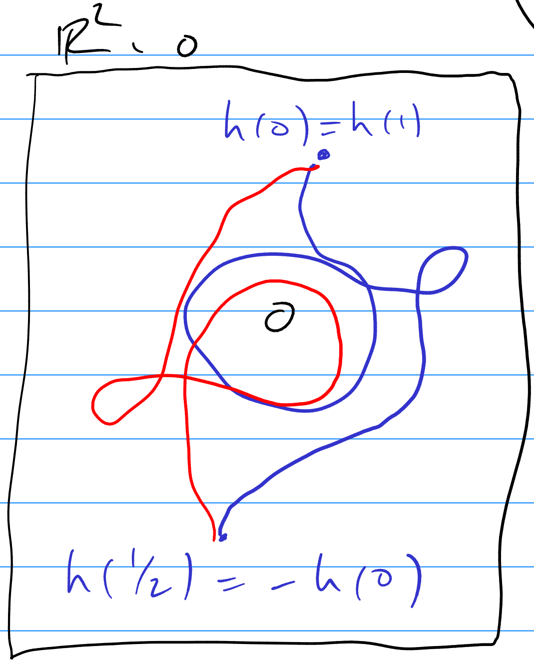


Let $h = g \circ \gamma : [0,1] \longrightarrow \mathbb{R}^2 \setminus \{0\}$

$h(0) = h(1)$, so h winds around the origin (some # of time)

claim it's an odd #





} winds $n + \frac{1}{2}$ times

Same $n+\frac{1}{2}$ times, rotated $180°$ {

winds $2n+1$ times in total

Now deform γ: let γ_s parametrize the parallel $z = s$, $0 \le s \le 1$

Let $h_s = g \circ \gamma_s$. Winding number is const as s varies.

But γ_1 is the constant map $\gamma_1(t) = (0,0,1)$ $\forall t$

so h_1 is const so winding # of $h_1 = 0$

not odd. $\rightarrow \leftarrow$ $\boxtimes$

Claim 3: $f : S^2 \to \mathbb{R}^3$ cont

then $\exists x \in S^2$ s.t. $f(x) = f(-x)$.

False: f could be the natural embedding

$S^2 \hookrightarrow \mathbb{R}^3$