Worksheet 6

Math 607, Connections and Characteristic Classes

Friday, January 29, 2021

In lecture we defined *associated bundles.* Let

$$\begin{tikzcd} G \arrow[r, hook] & P \arrow[d, "\pi"] \\ & X \end{tikzcd}$$

be a principal G-bundle, so G acts on P on the right, and acts freely and transitively on each fiber. If G acts on a space F on the left, then we get an associated F-bundle

$$\begin{tikzcd} F \arrow[r, hook] & E \arrow[d, "\varpi"] \\ & X \end{tikzcd}$$

where

$$E = P \times F \;/\; (pg, f) \sim (p, gf) \text{ for all } g \in G.$$

1. Given a connection on P, we get a parallel transport operator, which assigns to each piecewise smooth path $\gamma\colon [0,1] \to X$ a map from the fiber $\pi^{-1}(\gamma(0)) \cong G$ to $\pi^{-1}(\gamma(1)) \cong G$ that is equivariant for the action of G on the right.

   Given a parallel transport operator on P, write down a parallel transport operator on the associated bundle E.

2. (If you have time.) From that parallel transport operator on E, we could recover a splitting of

   $$0 \to T\varpi \to TE \to \varpi^* TX \to 0.$$

   Try to get that splitting directly from the splitting of

   $$0 \to T\pi \to TP \to \pi^* TX \to 0$$

   without using parallel transport operators.