

# Worksheet 9

Math 607, Homological Algebra

Monday, April 20, 2020

Let  $M$  and  $N$  be  $R$ -modules, and suppose we've chosen projective resolutions of  $M$  and  $N$  and a chain map lifting  $0: M \rightarrow N$  as shown:

$$\begin{array}{ccccccccc}
 \cdots & \longrightarrow & P_2 & \xrightarrow{d_2} & P_1 & \xrightarrow{d_1} & P_0 & \xrightarrow{\epsilon} & M & \longrightarrow & 0 \\
 & & \downarrow f_2 & & \downarrow f_1 & & \downarrow f_0 & & \downarrow 0 & & \\
 \cdots & \longrightarrow & P'_2 & \xrightarrow{d'_2} & P'_1 & \xrightarrow{d'_1} & P'_0 & \xrightarrow{\epsilon'} & N & \longrightarrow & 0
 \end{array}$$

1. Show that there is a map  $s_0: P_0 \rightarrow P'_1$  such that  $f_0 = d'_1 s_0$ .
2. Show that there is a map  $s_1: P_1 \rightarrow P'_2$  such that  $f_1 = d'_2 s_1 + s_0 d_1$ .