

Worksheet 8

Math 607, Homological Algebra

Friday, April 17, 2020

1. In lecture we let $R = k[x, y]$, $M = R/x$, and $N = R/(x, y)$, and we computed $\text{Tor}_*(M, N)$ using the free resolutions

$$R \xleftarrow{x} R \longleftarrow 0$$

and

$$R \xleftarrow{\begin{pmatrix} x & y \end{pmatrix}} R^2 \xleftarrow{\begin{pmatrix} y \\ -x \end{pmatrix}} R \longleftarrow 0.$$

Now compute $\text{Ext}^*(M, R)$ and $\text{Ext}^*(N, R)$. This is another way to see that $\text{proj dim } M = 1$ and $\text{proj dim } N = 2$.

2. Compute $\text{Ext}^*(M, N)$ and $\text{Ext}^*(N, M)$.
3. If you have time, check your answers in Macaulay2:

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R = QQ[x,y]
M = R^1/x
N = R^1/(x,y)

prune Hom(M, R^1)
prune Ext^1(M, R^1)
prune Ext^2(M, R^1)
-- et cetera
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