# Worksheet 7

Math 607, Homological Algebra

Wednesday, April 15, 2020

1. Let $R = k[x, y]$. Verify that the following is a free resolution of $R/(x, y)$:

$$R \xleftarrow{\left(\begin{smallmatrix} x & y \end{smallmatrix}\right)} R^2 \xleftarrow{\left(\begin{smallmatrix} y \\ -x \end{smallmatrix}\right)} R \longleftarrow 0$$

   I've written it "backwards" so you can multiply the matrices at a glance.

2. Let $R = k[x, y, z, w]$, and let $S = R/(xz + yw)$. Start to verify that the following is a free resolution of $S/(x, y)$:

$$S \xleftarrow{\left(\begin{smallmatrix} x & y \end{smallmatrix}\right)} S^2 \xleftarrow{\left(\begin{smallmatrix} y & z \\ -x & w \end{smallmatrix}\right)} S^2 \xleftarrow{\left(\begin{smallmatrix} w & -z \\ x & y \end{smallmatrix}\right)} S^2 \xleftarrow{\left(\begin{smallmatrix} y & z \\ -x & w \end{smallmatrix}\right)} S^2 \xleftarrow{\left(\begin{smallmatrix} w & -z \\ x & y \end{smallmatrix}\right)} S^2 \longleftarrow \cdots$$

   Notice that after the first term it's 2-periodic. Get as far as you can, and we'll revisit this example in lecture on Friday.