Worksheet 6

Math 607, Homological Algebra

Monday, April 13, 2020

1. In lecture we let $R = k[x, y, z, u, v, w]$,

$$A = \begin{pmatrix} 0 & x & y & z \\ -x & 0 & u & v \\ -y & -u & 0 & w \\ -z & -v & -w & 0 \end{pmatrix},$$

   and $M = \operatorname{coker}(A \colon R^4 \to R^4)$, and we analyzed the ranks of the fibers of M using the following Macaulay2 code:

```
R = QQ[x,y,z,u,v,w]
A = genericSkewMatrix(R,4)
radical minors(4,A) -- answer is a quadric hypersurface
radical minors(3,A) -- same
radical minors(2,A) -- answer is the origin
radical minors(1,A) -- same
```

   Do the same with $R = k[x, y, z]$,

$$A = \begin{pmatrix} 0 & x & y \\ -x & 0 & z \\ -y & -z & 0 \end{pmatrix},$$

   and $M = \operatorname{coker}(A \colon R^3 \to R^3)$.

2. If you have more time, let $R = k[x, y, z, w]$ and let M be the ideal

$$M = (xz - y^2, xw - yz, yw - z^2),$$

   which is the affine cone over the twisted cubic for those who know what that means. Analyze the ranks of the fibers of M using the following Macaulay2 code:

```
R = QQ[x,y,z,w]
M = module ideal(x*z-y^2, x*w-y*z, y*w-z^2)
C = resolution M
A = C.dd_1 -- a 3x2 matrix
-- now start taking minors of A as above
```