Worksheet 5

Math 607, Homological Algebra

Friday, April 10, 2020

1. Let R be a Noetherian local ring with maximal ideal $\mathfrak{m}$ and residue field $k = R/\mathfrak{m}$. Let N be a finitely generated R-module. Show that a map

   $$f\colon M \to N$$

   is surjective if and only if

   $$f \otimes 1\colon M \otimes k \to N \otimes k$$

   is surjective.

   Hint: Apply Nakayama's lemma to $\operatorname{coker} f$.

2. Show that it's not true with "injective" in place of "surjective":

   (a) Let $R = \mathbb{Z}_{(2)}$ and let f be the map of free modules $R \xrightarrow{\cdot 2} R$.

   (b) Let $R = k[x, y]_{(x,y)}$ and let f be $R \xrightarrow{\cdot x} R$.