

# Worksheet 4

Math 607, Homological Algebra

Wednesday, April 8, 2020

1. Let  $R$  be a commutative ring and  $P$  a projective  $R$ -module. Show that  $P_{\mathfrak{m}}$  is projective as an  $R_{\mathfrak{m}}$ -module for every maximal ideal  $\mathfrak{m} \subset R$ .

Hint: Your answer might involve tensor-hom adjunction.

[Update: Or just say that  $P$  is a direct summand of a free module.]

2. Let  $R$  be a commutative ring and  $P$  a finitely presented module that is *not* projective. Show that there is a maximal ideal  $\mathfrak{m} \subset R$  such that  $P_{\mathfrak{m}}$  is not projective as an  $R_{\mathfrak{m}}$ -module.

Hint: Use Eisenbud Proposition 2.10, which we proved in lecture, and model yourself on the proof we did in Friday's lecture with flatness.