Worksheet 21

Math 607, Homological Algebra

Wednesday, May 27, 2020

1. Sketh the line $x = y = 0$ and the point $x = y = z = 0$ in 3-space.
2. Let $R = k[x, y, z]$. Using the Koszul resolution
$$0 \to R \to R^3 \to R^3 \to R \to R/(x, y, z) \to 0,$$
verify that
$$\mathrm{Ext}^i_R(R/(x, y, z),\ R) = \begin{cases} 0 & i = 0 \\ 0 & i = 1 \\ 0 & i = 2 \\ R/(x, y, z) & i = 3. \end{cases}$$
3. For a finite resolution
$$0 \to A^{-n} \to \cdots \to A^{-1} \to A^0 \to N \to 0,$$
we have a second-quadrant spectral sequence
$$E_1^{p,q} = \mathrm{Ext}^q_R(M, A^p) \implies \mathrm{Ext}^{p+q}(M, N).$$
Using the Koszul resolution
$$0 \to R \xrightarrow{\binom{y}{-x}} R^2 \xrightarrow{(x\ y)} R \to R/(x, y) \to 0,$$
use this Koszul resolution to show that
$$\mathrm{Ext}^i_R(R/(x, y, z),\ R/(x, y)) = \begin{cases} 0 & i = 0 \\ k & i = 1 \\ k^2 & i = 2 \\ k & i = 3, \end{cases}$$
where k is short for $R/(x, y, z)$.

   Hint: The differentials on the E_1-page are what you think they are.
4. To compute $\mathrm{Ext}^i_R(R/(x, y, z),\ R/(x, y))$ directly, you could have applied $\mathrm{Hom}(-,\ R/(x, y))$ to the Koszul resolution in problem 2. Observe that this would have been harder.