Worksheet 2

Math 607, Homological Algebra

Wendesday, April 1, 2020

1. Consider the exact sequence of abelian groups
$$0 \to \mathbb{Z} \xrightarrow{2} \mathbb{Z} \to \mathbb{Z}/2 \to 0.$$
Apply the functors $\mathrm{Hom}(\mathbb{Z}/4, -)$, $\mathrm{Hom}(-, \mathbb{Z}/4)$, and $\mathbb{Z}/4 \otimes -$, and see where it fails to be exact. Do the same with $\mathbb{Z}/3$.

   If you're unsure about your calculation of Homs and tensor products, you can check in Macaulay2, for example:

```
Z2 = ZZ^1/ideal 2
Z4 = ZZ^1/ideal 4
prune(Z2 ** Z4)
prune Hom(Z4, ZZ^1)
```

   The `**` means tensor, and `ZZ^1` means a free module (whereas `ZZ` would be a ring).

2. Let k be a field, let $R = k[x, y]$, and consider the exact sequences of R-modules
$$0 \to R \xrightarrow{x} R \to R/x \to 0.$$
Apply Hom from and to the modules R/x, R/y, and $R/(x, y)$, and tensor with them, and see what happens with exactness. Again you can check your calculations in Macaulay2, for example:

```
R = QQ[x,y]
prune Hom(R^1/x, R^1/(x,y))
prune Hom(R^1/(x,y), R^1.x)
prune Hom(R^1/y, R^1)
```

3. Convince yourself that the following sequence of R-modules is exact, and play the same game:
$$0 \to R/y \xrightarrow{x} R/xy \to R/x \to 0.$$
Note that R/xy is different from $R/(x, y)$.