Worksheet 15

Math 607, Homological Algebra

Wednesday, May 6, 2020

In Example 17.3, Eisenbud considers the ring

$$R = k[x, y, z]/(x - 1)z,$$

and explains that x, $(x-1)y$ is a regular sequence in that order, but not in the other order $(x-1)y$, x, which is irritating. In lecture today we saw that this kind of thing cannot happen in a local ring.

1. In k^3, sketch the sets $(x-1)z = 0$ and $x = 0$ and $(x-1)y = 0$.

2. Consider the maximal ideal $\mathfrak{m} = (x, (x-1)y) = (x, y) \subset R$. Show that in the localization $R_{\mathfrak{m}}$, the sequence x, $(x-1)y$ is regular in either order.

3. Let $\mathfrak{n} \subset R$ be any other maximal ideal. Show that in $R_{\mathfrak{n}}$, the sequence x, $(x-1)y$ fails to be regular in any order for the silly reason that it generates the unit ideal.