Worksheet 14

Math 607, Homological Algebra

Monday, May 4, 2020

1. We know that $\mathbb{Z} \to \mathbb{Q}$ is flat. Show by example that it is not faithfully flat: that is, find a sequence

$$0 \to A \to B \to C \to 0$$

of Abelian groups that is *not* exact, but becomes exact when you tensor with $\mathbb{Q}$.

1′. Same with $k[x, y] \to k(x, y)$.

Hint: The Koszul complex

$$0 \longrightarrow R \xrightarrow{\binom{y}{-x}} R^2 \xrightarrow{(x\ y)} R \longrightarrow 0$$

is a nice example; understand why it works.

2. Show that $\mathbb{Z} \to \mathbb{Z}[\sqrt{-3}]$ is faithfully flat.

Hint: as a $\mathbb{Z}$-module, $\mathbb{Z}[\sqrt{-3}] \cong \mathbb{Z}^2$.

2′. Same with $k[x] \to k[x, y]/(y^2 = x^3 + x)$.

3. If you still have some time, show that if there is a faithfully flat map $R \to S$ then $\operatorname{glob\,dim} R \le \operatorname{glob\,dim} S$. Problems 1 and 1′ above show that you can't do without "faithfully."