Worksheet 13

Math 607, Homological Algebra

Friday, May 1, 2020

1. Let $R = k[x, y, z]$, and let $M = R^3$ with basis $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$. In lecture we discussed the Koszul complex

$$\Lambda^3 M^* \xrightarrow{\xi \lrcorner} \Lambda^2 M^* \xrightarrow{\xi \lrcorner} M^* \xrightarrow{\xi \lrcorner} R,$$

where $\xi = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} \in M$ is the Euler vector field.

Write down the 3×1, 3×3, and 1×3 matrices reperesenting the three maps in the complex, with the following choice of basis:

For M^*, take dx, dy, dz.

For $\Lambda^2 M^*$, take $dx \wedge dy$, $dx \wedge dz$, $dy \wedge dz$ in that order.

For $\Lambda^3 M^*$, take $dx \wedge dy \wedge dz$.

The entries of your matrices will all be $\pm x$, $\pm y$, $\pm z$, or 0. Be careful, get the signs right, don't accidentally transpose the 3×3 matrix, etc.

2. Open Macaulay2 and run the following code:

```
R = QQ[x,y,z]
xi = matrix{{x,y,z}}
K = koszul xi
```

The differentials `K.dd_1`, etc. should match what you got in #1.