Worksheet 11

Math 607, Homological Algebra

Monday, April 27, 2020

1. Convince yourself that the set in k^3 cut out by the equations $xy = xz = 0$ is the union of the x-axis and the yz-plane. Make a quick sketch. Mark the smooth points $(1, 0, 0)$ and $(0, 1, 0)$, and the singular point $(0, 0, 0)$.

2. Use Macaulay2 to study the self-Tors of the quotient fields corresponding to those three points, as follows:

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R = QQ[x,y,z]/(x*y,x*z)

m1 = ideal(x-1,y,z)
m2 = ideal(x,y-1,z)
m3 = ideal(x,y,z)

C = resolution(R^1/m1, LengthLimit => 6)
-- have a look at the differentials:
C.dd_1
C.dd_2
C.dd_3 -- etc

k = R/m1 -- this is a ring, not a module
C' = C ** k -- two stars means tensor
-- look at the differentials
C'.dd_1
C'.dd_2
C'.dd_3 -- etc

-- look at the ranks of the differentials
rank C'.dd_1
rank C'.dd_2
rank C'.dd_3
```

   Now use these ranks to compute (by hand) the dimensions of $\mathrm{Tor}_*^R(R/\mathfrak{m}_1, R/\mathfrak{m}_1)$.

   Do the same with $\mathfrak{m}_2$ and $\mathfrak{m}_3$.

   Next time we'll discuss this calculation in the context of localization.