Homework 1

Due Wednesday, September 26, 2018

You may work with other students on these problems, but you must write them up yourself, in your own words. Use pencil and double space (skip lines). If you type, use TEX, not Microsoft Word. Do not use the shorthand $\forall$ and $\exists$; you have time to write "for every" and "there is."

1. On Monday we saw several examples of metrics on $\mathbb{R}^2$, including the usual *Euclidean* or L^2 metric, which we'll denote here as $d_E(p, q)$, and the silly *SNCF* or *British Rail* or *LTD bus* metric:

$$d_*(p,q) = \begin{cases} d_E(p,q) & \text{if } p \text{ and } q \text{ lie on the same line through the origin, or} \\ d_E(p,0) + d_E(0,q) & \text{otherwise.} \end{cases}$$

   (a) Give an example of a sequence that converges in the Euclidean metric but not in the SNCF metric.

   (b) Prove that every sequence that converges in the SNCF metric converges in the Euclidean metric.

2. Let (X, d_X) and (Y, d_Y) be metric spaces, let $\{p_n\}$ be a sequence in X that converges to a point $\ell \in X$, and let $f\colon X \to Y$ be continuous at ℓ. Show that the sequence $\{f(p_n)\}$ in Y converges to $f(\ell)$.

3. What is one question you have about Monday's lecture?