Worksheet 16

Math 392, Abstract Algebra

Friday, February 19, 2021

Get out your cubes from the other day.

Let G be the group of rotations of the cube. Our convention is that multiplication in G works like composition of functions, so if g and h are two elements of G, then gh means g *after* h. We could have made the opposite choice, but we didn't.

1. Choose two adjacent faces of your cube and call them face 1 and face 2. Let h be an element of G that rotates face 1 by $90°$, and let g be an element that moves face 1 to face 2. Of the elements

$$g^{-1}hg \qquad ghg^{-1} \qquad h^{-1}gh \qquad hgh^{-1},$$

one of them rotates face 2 by $90°$. Which one is it? Try to get a feeling for why the answer makes sense.

2. Choose two adjacent vertices of your cube and call them vertex a and vertex b. Let h be an element of G that rotates vertex a by $120°$, and let g be an element that moves vertex a to vertex b. Of the elements

$$g^{-1}hg \qquad ghg^{-1} \qquad h^{-1}gh \qquad hgh^{-1},$$

one of them rotates vertex b by $120°$. Which one is it? Does this still make sense?

3. Let h be an element of G that rotates face 1 by $180°$, and let k be an element that rotates some edge by $180°$. Convince yourselves that there is no $g \in G$ such that $ghg^{-1} = k$.

4. Challenge: Let h be a $90°$ rotation as in problem 1. Convince yourselves that there is an element $g \in G$ such that $ghg^{-1} = h^{-1}$. How many such elements are there? Same with a $120°$ rotation as in problem 2.