Homework 8

Due Friday, February 26, 2021

1. For cubical dice, the standard way to label the faces is so that opposite faces add up to 7: so 1 is across from 6, 2 is across from 5, and 3 is across from 4. Actually there are two ways to do this, depending on whether 1, 2, and 3 go clockwise or counterclockwise around the vertex that they share.

   But if you relax this requirement, how many ways are there to label the faces of a die?

   Explain what this has to do with the cosets of H in G, where G is the symmetric group S_6 and H is the subgroup given by rotations of a cube.

2. Nick, Rachel, Gwen, and Alice (my family) are going to sing a four-part round like “Row, Row, Row Your Boat” or “Frère Jaques.” You might say there are $4! = 24$ ways to assign the parts: four choices for who goes first, then three for who goes second, and so on. But after they’ve been singing for a while, Nick–Rachel–Gwen–Alice will be indistinguishable from Rachel–Gwen–Alice–Nick, for example.

   How many ways are there to assign the parts, up to the notion of equivalence that this suggests?

   Explain what this has to do with cosets of H in G, where G is the symmetric group S_4 and $H \cong \mathbb{Z}_4$ is the cyclic subgroup generated by the permutation

$$\sigma = \begin{tikzcd} 1 \arrow[drrr] & 2 \arrow[dl] & 3 \arrow[dl] & 4 \arrow[dl] \\ 1 & 2 & 3 & 4. \end{tikzcd}$$

3. Let G be a group. Show that the map $\phi\colon G \to G$ given by $\phi(g) = g^2$ is a homomorphism if and only if g is Abelian. (Compare §6.2 #8.)

Also do **§6.2 #9 and 10**.