Worksheet 17

Math 391, Abstract Algebra

Wednesday, November 11, 2020

This is based on Shifrin §3.1 #2. For each field F and pair of polynomials $f, g \in F[x]$, use the Euclidean algorithm to find their greatest common divisor d:

(i) Assume that $\deg f \geq \deg g$; otherwise switch them. Divide f by g to get a quotient q_1 and a remainder r_1.

(ii) Divide g by r_1 to get a quotient q_2 and a remainder r_2.

(iii) Divide r_1 by r_2 to get a quotient q_3 and a remainder r_3.

(iv) Keep going until you get a remainder of zero. Then d is the last non-zero remainder that you got.

Just do as many of these as you have time for.

(a) $F = \mathbb{Q}$, $\quad f = x^4 + x^3 - x^2 - 2x - 2$, $\quad g = x^3 - 1$.

(b) $F = \mathbb{R}$, $\quad f = x^2 + (1 - \sqrt{2})x - \sqrt{2}$, $\quad g = x^2 - 2$.

(c) $F = \mathbb{C}$, $\quad f = x^2 + 1$, $\quad g = x^2 - i + 2$.

(d) $F = \mathbb{Q}$, $\quad f = x^2 + 2x + 2$, $\quad g = x^2 + 1$.

(e) $F = \mathbb{C}$ $\quad f = x^2 + 2x + 2$, $\quad g = x^2 + 1$.

Challenge: Also find polynomials $s, t \in F[x]$ such that $d = sf + tg$.