Worksheet 16

Math 391, Abstract Algebra

Friday, November 6, 2020

In lecture we saw how to find the roots of a cubic polynomial

$$x^3 + bx^2 + cx + d.$$

- Eliminate the quadratic term: substitute $x = z - \frac{b}{3}$ to get a "depressed cubic" of the form
$$z^3 + pz + q.$$
- Viète's trick: substitute $z = v - \frac{p}{3v}$, then multiply through by v^3 to get a quadratic in v^3.
- Find v^3 either by factoring the quadratic or by using the quadratic formula, then take cube roots to get v.

  (This is why we spent so much time taking cube roots earlier.)
- Substitute back to get z and then x.

Get started on §2.4 #6, which will be on the next homework.

6. Solve the following cubic equations:
   (a) $z^3 - 9z - 28 = 0$. (Answer: 4, $-2 \pm \sqrt{3}i$.)
   (b) $x^3 - 9x^2 + 9x - 8 = 0$. (Answer: 8, $\frac{1 \pm \sqrt{3}i}{2}$.)
   (c) $z^3 - 3z - 1 = 0$. (Answer: $2\cos 20°$, $2\cos 140°$, $2\cos 260°$.)