

Worksheet 15

Math 391, Abstract Algebra

Wednesday, November 4, 2020

Just do as many of these as you have time for. Recall that the cube roots of unity are 1,

$$\omega = e^{2\pi i/3} = \cos 120^\circ + i \sin 120^\circ = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,$$

and

$$\omega^2 = e^{4\pi i/3} = \cos 240^\circ + i \sin 240^\circ = -\frac{1}{2} - \frac{\sqrt{3}}{2}i,$$

and the fourth roots of unity are ± 1 and $\pm i$.

1. Find all three cube roots and all four fourth roots of 8. Plot them in the complex plane. Be prepared to show your picture to me or Elisa when we visit.
2. Same for 16.
3. Same for -8 .
4. Same for cube roots of $27i$.
5. Same for cube roots of $2 - 2i$. Your answer might involve $\sin 15^\circ$ and $\cos 15^\circ$; you could leave them as is, or if you're feeling energetic you could clean them up using the half-angle formulas

$$\cos^2(\theta/2) = \frac{1 + \cos \theta}{2} \qquad \sin^2(\theta/2) = \frac{1 - \cos \theta}{2}$$

with $\theta = 30^\circ$.

6. Same for cube roots of ω . Your answers will involve $\sin 40^\circ$ and $\cos 40^\circ$ and maybe some others that you can't clean up.
7. Same for fourth roots of $-6 - 2\sqrt{3}i$.