Worksheet 11

Math 391, Abstract Algebra

Friday, October 22, 2020

In lecture we considered the set of all subsets of the plane, and we made it into a ring by defining

$$X + Y = (X \setminus Y) \cup (Y \setminus X)$$

and

$$X \cdot Y = X \cap Y.$$

We found that 1 is the whole plane, and 0 is the empty set.

1. Show that the distributive property holds:

$$(X + Y) \cdot Z = X \cdot Z + Y \cdot Z.$$

   (Draw a Venn diagram to show what $(X + Y) \cdot Z$ looks like, and similarly with $X \cdot Z$ and $Y \cdot Z$ and $X \cdot Z + Y \cdot Z$.)

2. What are the units in this ring? That is, for which subsets X can you find another subset Y such that $X \cdot Y = 1$?

3. What are the zero-divisors in this ring? That is, for which subsets $X \neq 0$ can you find another subset $Y \neq 0$ such that $X \cdot Y = 0$?

4. It might seem more natural to define $X + Y = X \cup Y$ and $X \cdot Y = X \cap Y$, but this would not give a ring. Which axioms would fail, and which ones would still hold?

   (For a list of the ring axioms, refer to Wednesday's worksheet, or to the textbook.)