

Worksheet 1

Math 391, Abstract Algebra

Wednesday, September 30, 2020

Don't turn this in, just work through it with your colleagues. If you don't finish, that's ok, and if you finish early that's ok too. Elisa Bellah (the TA) and I will circulate through the breakout rooms to see how things are going, and there's also an "ask for help" button if you want to get our attention.

0. Introduce yourself to your colleagues. What other courses are they taking this term?
1. Here are a few lines of Pascal's triangle:

$$\begin{array}{ccccccc} & & & & 1 & & \\ & & & 1 & & 1 & \\ & & 1 & & 2 & & 1 \\ & 1 & & 3 & & 3 & & 1 \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

Write out a few more lines. Stop when you start to get 3-digit numbers.

2. Write out $(x + y)^5$ using the binomial theorem. Don't work hard.
3. Find $\binom{8}{3}$, first by looking it up in Pascal's triangle, then using the formula

$$\binom{n}{k} = \frac{n(n-1) \cdots (n-k+1)}{k(k-1) \cdots 1}.$$

Notice that there are k terms on both the top and bottom.

4. Six people sit down to eat dinner together, say "cheers," and clink their glasses together in pairs. How many clinking sounds are there?
5. What is $1 + 2 + 3 + \cdots + 100$? What does this have to do with the previous problem?
6. Challenge: What does the binomial theorem say about $(1 + x)^{1/2}$?