# Practice Final

December 6, 2015

Use your own notebook paper. Start each problem on a new sheet of paper. I recommend that you use pencil rather than pen. You may use a scientific calculator – one that can take square roots, natural logs, etc., but not derivatives and integrals. **I will provide calculators for the real exam.**

**1.**

(a) $\displaystyle\int \frac{1}{2-3x}\,dx$ is which of the following?

| | |
|---|---|
| $\ln(2-3x)+C$ | $-\ln(2-3x)+C$ |
| $3\ln(2-3x)+C$ | $-3\ln(2-3x)+C$ |
| $\frac{1}{3}\ln(2-3x)+C$ | $-\frac{1}{3}\ln(2-3x)+C$ |
| $2\ln(2-3x)+C$ | $-2\ln(2-3x)+C$ |
| $\frac{1}{2}\ln(2-3x)+C$ | $-\frac{1}{2}\ln(2-3x)+C$ |

(b) $\displaystyle\int \frac{1}{(2-3x)^2}\,dx$ is which of the following? Why?

| | | | |
|---|---|---|---|
| $\ln\left((2-3x)^2\right)+C$ | $-\ln\left((2-3x)^2\right)+C$ | $\frac{1}{2-3x}+C$ | $-\frac{1}{2-3x}+C$ |
| $3\ln\left((2-3x)^2\right)+C$ | $-3\ln\left((2-3x)^2\right)+C$ | $3\cdot\frac{1}{2-3x}+C$ | $-3\cdot\frac{1}{2-3x}+C$ |
| $\frac{1}{3}\ln\left((2-3x)^2\right)+C$ | $-\frac{1}{3}\ln\left((2-3x)^2\right)+C$ | $\frac{1}{3}\cdot\frac{1}{2-3x}+C$ | $-\frac{1}{3}\cdot\frac{1}{2-3x}+C$ |
| $2\ln\left((2-3x)^2\right)+C$ | $-2\ln\left((2-3x)^2\right)+C$ | $2\cdot\frac{1}{2-3x}+C$ | $-2\cdot\frac{1}{2-3x}+C$ |
| $\frac{1}{2}\ln\left((2-3x)^2\right)+C$ | $-\frac{1}{2}\ln\left((2-3x)^2\right)+C$ | $\frac{1}{2}\cdot\frac{1}{2-3x}+C$ | $-\frac{1}{2}\cdot\frac{1}{2-3x}+C$ |

**2. (Based on §6.3 #16.)**

(a) Sketch the region in the plane bounded by the parabolas $y=x^2$ and $y=2-x^2$. Find the coordinates of the points where the two curves meet, and the points where they meet the $y$-axis.

(b) Sketch the solid obtained by revolving the region around the $y$-axis.

(c) Find the volume of the solid using the shell method.

(d) Find the volume of the solid using the disc method. Hint: It may be convenient to find the volume of just the top half or the bottom half, and then double it.

If your answers to (c) and (d) disagree, go back and find your mistake, or at least indicate that you noticed.

**3. (§7.4 #13. Seasonal.)** A turkey is taken out of the oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F. Newton's law of cooling states that the turkey cools at a rate proportional to the difference between its temperature and the temperature of the room.[1]

(a) If the temperature of the turkey is 150°F after half an hour, what will it be after 45 minutes?

(b) When will the turkey have cooled to 100°F?

**4. (Based on an example we did in lecture.)** According to Toricelli's law, water drains from a tank at a rate proportional to the square root of the depth. A spherical tank of radius 1 meter was initially full of water and took an hour to drain. At what time was the tank half full?

Let's all choose the same variables: let $t$ be the time in minutes, $y$ the depth of the water in meters, and $V$ the volume of water in cubic meters.

(a) Draw a picture of the tank partly filled with water, but not exactly half full. Label $y$.

(b) What is the area of the surface of the water when the depth is $y$? Hint: The surface is a circle; start by finding its radius. It may help to draw a cross-section of the tank and some kind of triangle.

(c) Write a differential equation expressing Toricelli's law. Hint: The left-hand side is $dV/dt$; the right-hand side involves $y$ and a constant $k$.

(d) You could find $V$ as a function of $y$, or $y$ as a function of $V$, but it would be a lot of work. Instead, observe that $dV/dy$ is the area you found in part (b) – why is this true? Then observe that

$$\frac{dV}{dt} = \frac{dV}{dy} \cdot \frac{dy}{dt}$$

by the chain rule, so you can replace $dV/dt$ in your equation from part (c) with your new expression for $dV/dy$, times $dy/dt$, to get a differential equation involving only $y$.

(e) Take your equation from part (d), separate variables, and integrate. But do not try to solve for $y$.

(f) What is $y$ when $t = 0$? When $t = 60$? Use these facts to find $k$ and $C$. They will be a little messy, but not horribly so.

(g) What is $y$ when the tank is half full? Find $t$ corresponding to that value $y$. It should be a little less than 30 minutes, because the water flows faster at first and slower later on.

[1] If you have actually cooked a turkey you know that the breast, thigh, and stuffing, etc. will be at different temperatures, and the internal temperature will continue to rise for a while because the outside of the turkey is hotter than the inside. Moreover the turkey will warm the air around it, slowing the cooling, unless there's a draft. For extra credit, discuss whether it is reasonable to neglect all this.