

Practice Midterm 1

October 23, 2015

Use your own notebook paper. Start each problem on a new sheet of paper. I recommend that you use pencil rather than pen. No calculators, notes, or cheating.

1.

- (a) Sketch the region in the xy -plane whose area is given by $\int_0^1 \sqrt{4-x^2} dx$.
- (b) Evaluate the integral geometrically by thinking about triangles and wedges of circles.
- (c) Evaluate the integral algebraically by substituting $x = 2 \sin t$. Does your answer agree with your answer to part (b)? Hint: You will need the half-angle formula $\cos^2 t = \frac{1+\cos 2t}{2}$. Further hint: Don't forget to change bounds: if $2 \sin t = 1$ then $\sin t = \frac{1}{2}$, so $t = \dots$

2. Evaluate $\int x\sqrt{x-1} dx$ in two ways:

- (a) By substituting $u = x - 1$. Hint: Then $x = u + 1$.
- (b) By parts, letting $u = x$ and $dv =$ the rest.
- (c) Your answers to parts (a) and (b) look different. Show that they are the same. Hint: Factor out $(x-1)^{3/2}$ and clean up what's left.

3.

- (a) What is $\lim_{t \rightarrow \infty} e^t$? What is $\lim_{t \rightarrow \infty} e^{-t}$? What is $\lim_{t \rightarrow \infty} te^{-t}$? Hint: Write the latter as $\lim_{t \rightarrow \infty} \frac{t}{e^t}$, which is of the form $\frac{\infty}{\infty}$, so you can use l'Hôpital's rule.
- (b) Find $\int te^{-t} dt$. Hint: Integrate by parts.
- (c) Find $\int_0^\infty te^{-t} dt$. Hint: Recycle your answer to parts (a), (b), and especially (c).