

Midterm Exam 1

October 30, 2015

Use your own notebook paper. Start each problem on a new sheet of paper. I recommend that you use pencil rather than pen. No calculators, notes, or cheating. Each part is worth 10 points.

1.

- (a) Sketch the region in the xy -plane that lies above the x -axis and below the parabola $y = 4 - x^2$.
- (b) Find the area of the region.

2. Evaluate $\int \frac{x}{1-x^2} dx$ in three ways:

- (a) By substituting $u = 1 - x^2$.
- (b) By partial fractions: Factor $1 - x^2$ as $(1+x)(1-x)$, then find numbers A and B such that

$$\frac{x}{(1+x)(1-x)} = \frac{A}{1+x} + \frac{B}{1-x},$$

then integrate the right-hand side.

- (c) By substituting $x = \sin t$. Hint: You may need to recall that $\frac{\sin t}{\cos t} = \tan t$ and $\int \tan t dt = \ln(\sec t) + C = -\ln(\cos t) + C$.
- (d) Your answers to parts (a), (b), and (c) look different. Show that they are the same.

3.

- (a) Evaluate $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$.

Hint: This is of the form $\frac{\infty}{\infty}$, so you can use l'Hôpital's rule.

- (b) Evaluate $\int x^{-2} \ln x dx$.

Hint: Integrate by parts, letting $u = \ln x$ and $dv =$ the rest.

- (c) Evaluate $\int_1^\infty x^{-2} \ln x dx$. Hint: Recycle your answers to parts (a) and (b).

- (d) Substitute $x = e^t$ into the integral from part (c) and clean it up to get $\int_0^\infty te^{-t} dt$. On the practice exam you found that the latter equals 1. Does this agree with your answer to part (c)?