Homework 3

Due Friday, October 15, 2021

Math 205

1. On the board we saw that

$$\pi = 3.141\ldots = 3 + \frac{1}{7.062\ldots} = 3 + \frac{1}{7 + \frac{1}{15.996\ldots}}$$
$$= 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1.003\ldots}}} = 3 + \frac{1}{7 + \frac{1}{15 + \frac{1}{1 + \frac{1}{292.634\ldots}}}}$$

   Write a program that takes π and prints out the sequence of integers 3, 7, 15, 1, 292, ... continuing for at least 20 steps.

   You'll want to use `int(x)` rather than `round(x)`, because you want $15.996\ldots$ to turn into 15 rather than 16.

2. Python's built-in π is only accurate to 15 decimal places, so you'll eventually run into rounding errors. For greater precision, use the `decimal` package:

```
from decimal import *
x = Decimal("3.14159265358979323846")
```

   Better yet, search online for "digits of pi" and get 30 or 40 digits.

   Run your program again with this better approximation of π. At what step do you start to see different answers than you saw in #1?

3. Run your program on e rather than π. Do you notice any patterns?

   You can use `from math import e` as we did last week, or for greater precision you can use

```
x = Decimal(1).exp()
```

4. Choose some two-digit numbers and run your program on their square roots. Do you notice any patterns? Also check out $\sqrt{2}$, $\sqrt{3}$, and $\frac{1+\sqrt{5}}{2}$.

5. Pick two smallish integers m and n, and find a real number x such that

$$x = m + \cfrac{1}{n + \cfrac{1}{m + \cfrac{1}{n + \cfrac{1}{\ddots}}}}.$$

   Feed your answer into your program to check that you succeeded.

   Hint: Convince yourselves that such a number should satisfy

$$x = m + \frac{1}{n + \frac{1}{x}}.$$

   Then figure out how to solve that equation.

6. Run your program on some *cube* roots of two- or three-digit numbers. Do you notice any patterns?

7. Optional challenge problem: On the board we saw that rounding down the decimals in #1 gives the sequence of "unusually good" rational approximations to π that we found last week:

$$3 \qquad 3 + \frac{1}{7} = \frac{22}{7} \qquad 3 + \frac{1}{7 + \frac{1}{15}} = \frac{333}{106} \qquad \text{etc.}$$

   Can you adapt your program from #1 so that it also prints this sequence of fractions?

   You might want to use the `fractions` package, which works like this:

```
from fractions import *
x = Fraction(22, 7)
print(x)
```