

Worksheet 8

Wednesday, November 6, 2019

Math 205

1. You're probably used to identifying a real number with its decimal representation: if a is a number between 0 and 1, you can write

$$a = 0.a_1a_2a_3a_4a_5\dots,$$

where $a_0, a_1, \dots$ are integers between 0 and 9. Suppose that b is another number between 0 and 1, and that its decimal expansion is

$$b = 0.b_1b_2b_3b_4b_5\dots$$

Explain clearly how to find the decimal expansion of $a + b$. For example, if I want to know the 20th digit of $a + b$, can you explain to me how to find it? This may sound obvious, but you'll find there are some subtleties.

2. Explain clearly how to find the decimal expansion of $a \cdot b$.

3. Many of you have wanted to talk about a number

$$x = 0.00000 \dots 1,$$

with infinitely many zeros before the 1. But you probably haven't thought through all the implications of having such a number. What is $x/10$? $10x$? x^2 ? $1/x$? Can you make a number system that includes this number and has all the properties you want? (I'm not saying you can't, but it certainly isn't obvious.) How does this relate to our discussion in weeks 2 and 3?