

Worksheet 7

Monday, November 4, 2019

Math 205

1. You have been assigned into groups of three for the week. Write down your colleagues' names and email addresses.
2. A number is called *rational* if it can be written as p/q , where p and q are integers and $q \neq 0$. The rational numbers are “dense” in the real numbers: that is, between any two real numbers x and y there is a rational number p/q . Figure out why this is true, and explain it clearly.

3. A real number that is not rational is called *irrational*. Favorite examples include $\sqrt{2}$ and π . The irrational numbers are also dense: between any two real numbers x and y there is an irrational number z . Figure out why this is true, and explain it clearly.

4. What is the next rational number after π ?

5. You're probably used to identifying a real number with its decimal representation: if a is a number between 0 and 1, you can write

$$a = 0.a_1a_2a_3a_4a_5\dots,$$

where $a_0, a_1, \dots$ are integers between 0 and 9. Suppose that b is another number between 0 and 1, and that its decimal expansion is

$$b = 0.b_1b_2b_3b_4b_5\dots$$

Explain clearly how to find the decimal expansion of $a + b$. This may sound obvious, but you'll find there are some subtleties.